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Mesh Generation Methods and Moving Mesh Generation Using Developed Program

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Abstract:- One of the most commonly used methods in numerical solution of partial differential equations is the finite element method. In the finite element method, the region to be analyzed is divided into sub-sections called solution regions provided that the boundaries of the region are determined. This subdivision method depends on the type of differential equation to be solved. A variety of solution network production techniques are used to subdivide the solution region. By selecting the appropriate method, the solution region is divided into sub-compartments to ensure that the solution is faster and more accurate. The classical finite element method gives accurate results when instant analysis is performed on the solution area. However, in cases where partial differential equations change with time and solution network changes regionally, it is useful to use moving finite element method instead of classical finite element method. The use of a moving finite element method allows analysis to be carried out only in varying regions of the solution network to ensure rapid results. In this study, two dimensional solution network production techniques are mentioned. With the help of the developed program, regional changes on the solution network are explained in detail. As an application, C ++ based software was implemented.

Index terms: Mesh Generation Methods , Finite Element Method, Moving mesh generation

I. INTRODUCTION

Most scientists working with the physical environment face two important issues. These are the mathematical formulation of the physical environment and the numerical analysis of the mathematical model. The mathematical model of the physical environment often requires knowledge of basic mathematics and physics. Most of the expressions in the mathematical formulation are of the type of differential equations related to the system. The solution of these differential equations gives information about the behavior of the physical system being studied at any time [1]. For most problems it is not difficult to derive the corresponding equation. However, it is not always possible to obtain complete solutions of these equations. In this case, instead of finding a complete solution, approach methods are tried to be solved with a certain error. In the finite difference approximation of the differential equation, subsequent derivatives are represented by difference segments. Thus, the boundary conditions of the given physical environment are forced, the resulting algebraic equation is solved and the result is obtained. In variational methods, the equation is transformed into a weighted integral form and then it is assumed that the approximate solution on the domain is linearly connected [2]. The finite element method overcomes the disadvantages of conventional variational methods. The method uses the following steps for a simple problem:

A. Finite Element Solution Network Of The Given Region

This step may cause a delay until the finite element equation formulation is finished. In continuity problems, the field variable is a function of the points in the solution region. Thus, the field variable has an infinite number of values, and the problem is an infinite number of unknowns. In order to reduce the problem to a finite number of unknown states, as a first step, the solution region is divided by a finite number of elements. The elements are triangular, quadrangular, hexahedron, isoparametric, etc. shapes and different sizes. This selection is made taking into account the problem being one, two or three dimensional and the boundaries of the material or device under examination. Triangular elements provide convenience in irregular shapes and beautiful arrivals. These elements easily adapt to the boundary surfaces. Thus, the solution region can be divided by elements of desired size and frequency to change the number of nodes and thus the number of solution equations. After dividing these elements, the appropriate node numbers and element numbers are given to the elements [3].

B. Deriving Shape Functions For All Element Types On The Solution Network

Using the steps that determine each element, the type of the shape function showing the change of the field variable on or within that element is selected. The field variable may be scalar, vectorial or a high-grade tensor. Often polynomials are selected for the field variable. Because it is easy to take their integrals and derivatives. Polynomials are linear, quadratic, cubic, etc. structure. Appropriate boundary conditions of the system are determined.

C. Finding Element Properties

Finding matrix equations that define the properties of each element constitutes the third step. Some of the approaches used for this are; direct approach, variational approach, Weighted Residues approach, and Energy Balance Approach.

D. Combining Element Properties

To find the properties that model the whole system, the individual features of the elements are combined and the boundary conditions of the problem are placed in the system and the resultant equation system is made ready for the solution.

E. Solution Of The Equation System

Typical dependent variable;

$$u = \sum_{i=1}^{n} u_i \psi_i \quad (1)$$

$$\left[K^e\right]\left\{u^e\right\} = \left\{F^e\right\}$$

The element equation in the form is obtained. The *K* matrix is the so-called main matrix.

F. ψ_i Element Interpolation Function Selection And Derivation

The resultant equation system obtained in the fourth step is solved by using various methods to calculate the unknown values of the field variable. If the equations are linear; Direct methods such as Gauss Elimination, Cholesky Decomposition or iterative methods such as Gauss-Seidel, SOR (Successive Over Relaxion) may be used. If the equations are nonlinear, it is more difficult to solve them and iterative methods such as Newton-Raphson are used.

G. Other Required Calculations

The solution results of the equation system can be used to calculate some important parameters of the system. For example, the solution results of the Laplace equation, which provides an examination of the electric fields, give the potential distribution of the system. If desired, field strengths, stored energy, and capacity can be calculated using the potential values of the nodes [4].

To make accurate and reliable calculations with the finite element method, the finite element solution network of the solution region must be established. In the next section, solution network production techniques will be discussed.

II. Mesh Generation Algorithms

In this section, the mesh generation techniques used in finite element mesh production in two dimensions as both surface and volume elements are explained. As is known, the reason for the production of the mesh is to prepare data for analysis. In this section, mesh generaiton techniques that are suitable for adaptive structures used in finite element mesh products will be mentioned. These methods can be classified as follows [5]:

A. Two-dimensional triangular solution mesh:

a) Two-dimensional Delaunay triangulation
b) Advancing facade method.
c) Mesh generation using trajectory lines.
d) Edge splitting technique by sub-splitting.

B. Two dimensional rectangular solution mesh:

a) Direct methods.
b) Indirect methods.

A. Two-Dimensional Triangular Solution Mesh

This section will provide information about solution mesh production techniques commonly used in triangular solution mesh production.

a) Two-Dimensional Delaunay Triangulation

It is the most commonly used triangulation method. The method was first introduced by Russian mathematician Boris Delaunay [6] in 1934. The method itself is not a direct triangulation for the production of a solution network but is based on the principle of joining existing points. Although the idea of Delaunay has been in the literature for a long time, it was first used by Lawson [7], especially in the production of solution networks. It is not a direct triangulation method but is based on producing triangles at the boundaries until there are no intersections and the Delaunay condition is met. The Delaunay requirement is to triangulate the points to be joined to form at least one triangle in two dimensions and the corners of the formed triangle to obtain a circle. For this purpose, the distribution of the points must be made to form the Voronoi diagram or this condition should be sought at the points to be joined. *Fig. 1* shows Voronoi and Delaunay for seven points. The dotted line is Voronoi, and the continuous line is Delaunay.

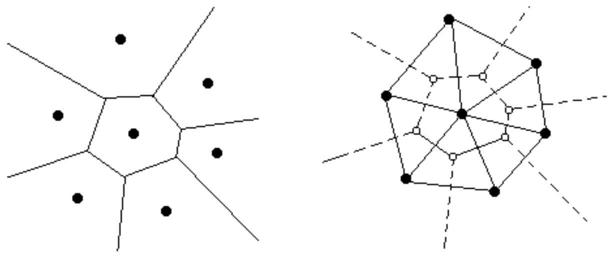


Fig 1. Voronoi and Delaunay edges for seven points

Fig. 2 shows two triangulation methods, Delaunay and non-Delaunay. The method first triangulates using discrete points in the boundary. The points to be added later are required to be placed appropriately and meet the Delaunay requirement. Various techniques have been proposed for this process.

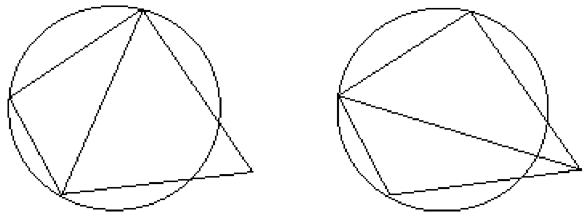
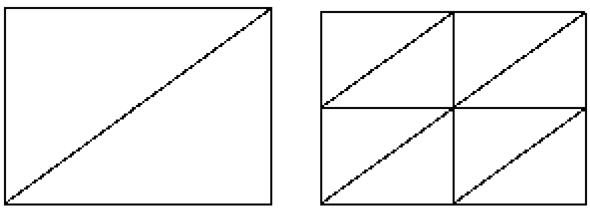


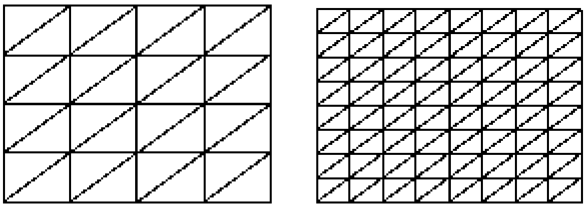
Fig. 2 a) Delaunay triangulation b) Delaunay non-triangulation

b) Edge Splitting Technique By Sub-Splitting

In this technique, initially, the coarse partitioning of the area where the solution network is to be produced is roughly segmented by a manual or automatic dividing program and then divided into smaller segments. This process is known as a subdivision in computer graphics [13]. The necessary condition for subdivision is a pre-division. *Fig. 3* shows a simple rectangular structure in which the subdivision technique is applied.



a) **b)**



c) **d)**

Fig. 3 Double subdivision technique, a is the original figure b,c,d are subdivison figures.

The process is to divide each edge into two equal parts and obtain four triangles from one triangle. This method is called a binary subdivision. The advantage of this method is that the integrity of object boundaries can be maintained. In this way, as many partitioning as desired on the entire solution network will be provided. The method is capable of providing both regional and general control. *Fig. 4* shows a more regionally generated solution mesh.

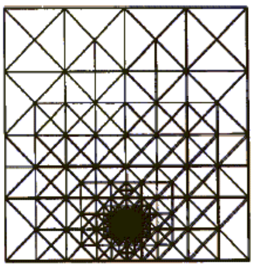


Fig. 4 Regional controlled solution mesh produced by subdivision technique

B. Two Dimensional Rectangular Solution Mesh

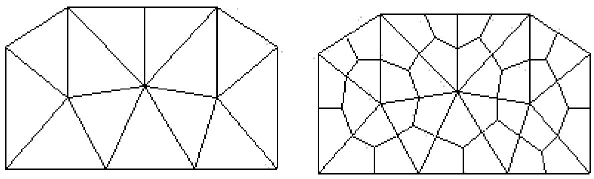
Mapping (mapping) and drawing method and rectangular solution network production techniques are the most commonly used solution network production techniques [14]. The general feature of this method is that only structural solution networks can be produced. However, non-structural solution networks can be produced indirectly by changes in element dimensions. Non-structural rectangular solution network production techniques are divided into two main groups; These are direct and indirect methods.

a) Direct Methods

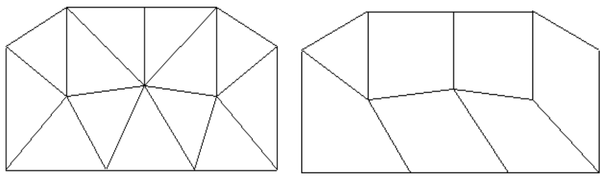
It is possible to divide the methods directly into two main groups. First; The problem is that the environment is divided into simple regions. The other is that the location of the nodes and elements can be produced in a manner similar to the progressive façade method. The quadrilateral tree method and the quadrilateral solution network production technique fall into the first group. After initial partitioning in two-dimensional space, sub-partitioning is performed until the integrity of the boundaries is achieved. Talbert has given a method that uses the self-iterative algorithm for subdivision. Zhu [15] emphasized the formation of rectangles starting from the borders with the help of the advancing front method.

b) Indirect Methods

The basis of the indirect methods is to divide the area of the solution network into triangles first and then to form rectangular structures by combining or dividing the triangles. *Fig. 5* shows a rectangular solution web thus produced.



a)



b)

Fig. 5 Indirect rectangular solution network generation a) forming three rectangles from a triangle b) Combining two triangles and forming a rectangle

III. MOVING SOLUTION MESH GENERATION

Many numerical methods have been developed for the solution of partial differential equations and each of these methods has its own advantages and disadvantages. These numerical methods are selected using information about the problem type. In particular, these numerical methods are used to solve nonlinear problems such as the law of the conservation of mass containing moving facades and impacts. In such problems, the solution in some regions may have high resolution; in some regions it is less soluble. This required resolution can be achieved by increasing the number of nodes or by using adaptive methods. Thus, by increasing the number of nodes in the regions where the solution changes rapidly, the solution is made faster and more accurately. In our program, the regions where the solution changes rapidly can be divided into more triangular elements and finite element analysis can be achieved faster and more accurately. *Fig. 6* is the flow chart of the program we developed.

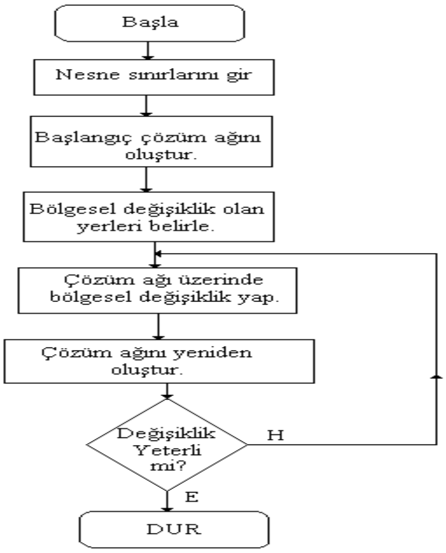
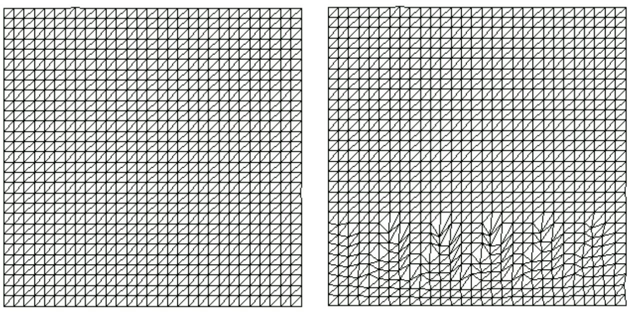


Fig. 6 Flow chart of the developed program

Fig. 7 shows the change along a sine line on the solution network obtained with the help of the program developed. The number of grids used here are 841 quadrangular and 1682 triangular elements. Here the variation around the part of the $y = \sin(x)$ equation of a certain thickness is shown.



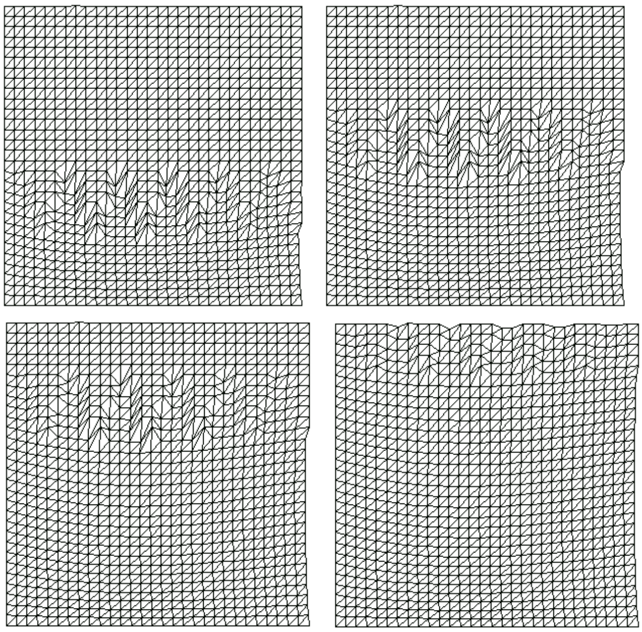


Fig. 7 Change along a sine line on the generated solution mesh

Fig. 8 shows the change in a circular region on the solution network obtained with the help of the program developed. The number of grids used here are 841 quadrangular and 1682 triangular elements.

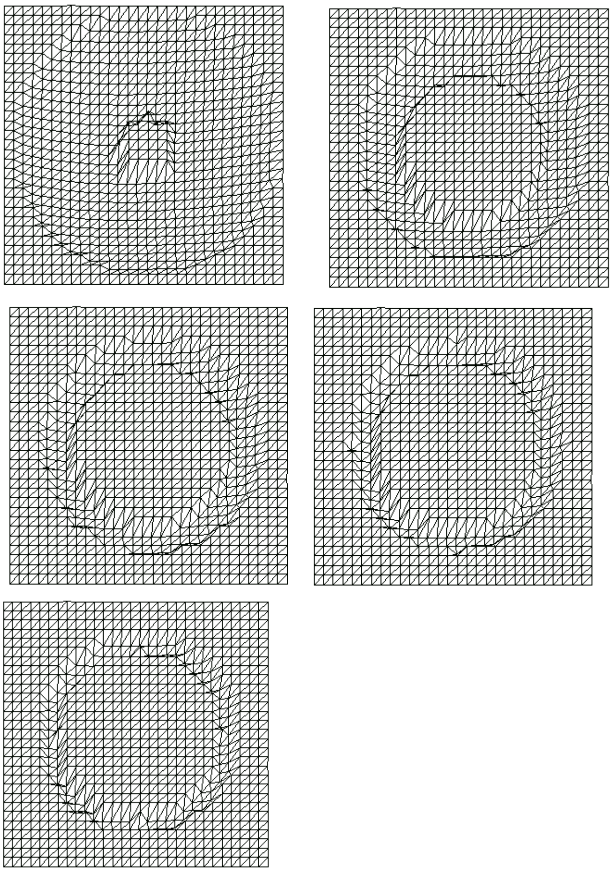


Fig. 8 Circular change over the generated solution mesh

IV. RESULTS

In this study, solution mesh production methods are mentioned and program solution mesh production examples are given. In this way, the more accurate and faster solution can be achieved by increasing the number of elements in the region where differential equation changes rapidly. Especially when performing finite element analysis of electric machines, more accurate and faster results will be achieved if finite element analysis is carried out by obtaining a mobile solution network in regions where electric field strength and magnetic flux change rapidly.

With the help of the program developed in the following studies, a finite element analysis can be made by producing a moving solution mesh of a time-varying differential equation and the results can be compared with classical finite element analysis.

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